The Role of Porosity in Filtration
Part 3: Variable-pressure–Variable-rate Filtration

Virtually all filtration literature has been concerned with constant rate or constant pressure with greater emphasis on the latter. In contrast to these types of operations, industrial filtrations involving centrifugal pumps are accomplished under variable-pressure–variable-rate conditions. In spite of its importance virtually no work has been reported in connection with variable-rate–variable-pressure filtration. Formulas developed for constant pressure and constant-rate filtration are not in general applicable to operations effected by centrifugal pumps. Methods solving variable-pressure–variable-rate filtration problems are presented.

A method of determining average filtration resistance as a function of compressive pressure under variable-pressure–variable-rate conditions is discussed, and formulas for determining point filtration resistance from data for average resistances are presented.

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In previous articles in this series, graphical procedures for constant-pressure and constant-rate filtration (*6*) and analytical equations for constant-rate filtration (*7*) were developed. Variation in porosity throughout the filter bed and relationships of porosity to filtration resistance served as a basis for predicting pressure and volume as a function of time. In this paper emphasis is placed on the solution of filtration problems in which neither rate nor pressure is maintained constant. While porosity does not explicitly enter the equations presented, it is an important factor in determining the filtration resistance.

Current development of the mathematical art of filtration depends on a number of assumptions which have been only indirectly substantiated. Among the more important postulates are

1. Ultimate values of porosity are attained instantaneously. This assumption is probably valid for filtrations in which pressure increases slowly.
2. There is point contact between particles. The basic equation, $p_x + p_s = p$, where p_x is the hydraulic pressure, p_s the solid compressive pressure, and p the applied filtration pressure, depends upon the postulate of point contact.
3. The point filtration resistance of a given solid is determined by the porosity, which in turn depends upon the compressive solid pressure p_s.
4. The porosity or specific filtration resistance determined under a given mechanical loading p_s in a compression-permeability cell is the same as the porosity or resistance at a point in a filter cake where the solid pressure (computed by $p_s = p - p_x$) is the same as the mechanical loading in the compression-permeability cell.
5. Flow is viscous. In general investigators have not concerned themselves with turbulent flow.

As the mathematical theory largely rests on item 4, it is perhaps the most important assumption. While the results of Grace (*2*) and Kottwitz (*4*) lend validity to the postulate, complete verification must await simultaneous experimental determination of porosity and hydraulic pressure variations within an actual filter cake.

TYPES OF FILTRATION

For purposes of mathematical treatment, filtration operations are classified according to the variation of the pressure and flow rate with time. Generally, the pumping mechanism determines the flow characteristics and serves as a basis for division into the following categories:

1. Constant pressure filtration. Actuating mechanism is compressed air maintained at a constant pressure.
2. Constant rate filtration. Positive displacement pumps of various types are employed.
3. Variable-pressure–variable-rate filtration. The use of a centrifugal pump results in the rate varying with the back pressure on the pump.
4. Stepped pressure. For experimental purposes, it is possible to manually increase pressure during a filtration and simulate various pumping conditions.

Flow rate vs. pressure characteristics for the four types of filtration are illustrated in Figure 1. Arrows drawn on the curves point in the direction of increasing time. The constant pressure curve is represented by a vertical line, the downward arrows indicating that the rate decreases with time. Drawn horizontally, the constant-rate filtration curve has arrows pointing to increasing pressure with time. The rate for a filter actuated by a centrifugal pump will follow the downward trend of the variable-pressure–variable-rate curve. Depending upon the characteristics of the centrifugal pump, widely differing curves may be encountered. If the first portion of the curve is nearly flat, the pump will produce a filtration which is almost at constant rate. The dotted curve is approximately equivalent to a filtration carried out first at constant rate and then at constant pressure.

In laboratory practice, it is possible to duplicate any desired conditions with stepped-pressure operation (*3*). If an experimental press is instrumented for determining flow rate, the pressure may be modified to give any predetermined rate characteristics.

In the literature, by far the greatest attention has been focused on constant-pressure filtration. While a small amount of effort has been directed toward the more significant constant-rate filtration, the industrially important area of variable-pressure–variable-rate filtration has been virtually untouched. The remainder of this paper will be devoted to developing methods for predicting pressure-volume-time relationships in variable-pressure–variable-rate operation. In addition, the author will discuss his belief that this type of filtration is more simple than either constant-pressure or constant-rate filtration for determination of cake characteristics.

BASIC EQUATIONS

As liquid flows frictionally through a bed of compressible solids, viscous drag on the particles produces an accumulative compressive pressure which causes the porosity to decrease as the supporting medium is approached. For point contact between solids, the following relation holds (*1, 6, 7*):

$$dp_s + dp_x = 0 \qquad (1)$$

where p_x is the hydraulic pressure at a distance x from the surface of the cake

TABLE 1

Filtration Pressure ψ, lb./sq. in.	Pressure at medium ψ_1 lb./sq. in.	Pressure difference, lb./sq. in.	Rate of filtration q_g, gal./(min.)(sq. ft.)	Value of integral Eq. (15*b*)	v_g from (15*b*) gal./sq. ft.
5	5	0	0.540	0	0
10	4.9	5.1	0.506	52.5(10^{-12})	8.4
15	4.6	10.4	0.470	72.0	12.5
20	4.2	15.8	0.434	88.0	16.5
25	3.8	21.2	0.391	102.5	21.3
30	3.4	26.6	0.336	115.5	28.1
35	2.5	32.5	0.263	129.0	39.9
40	1.7	38.3	0.172	140.0	65.2

For Parts 1 and 2, see references 6 and 7.

(Figure 2) and p_s is the solid compressive pressure. Integration of (1) yields

$$p_s + p_x = p \qquad (2)$$

where p is the applied pressure at the surface of the cake.

The fundamental equation relating the pressure gradient to the rate of flow is [reference 2, Equation (1)]

$$g_c \frac{dp_x}{dw_x} = \alpha_x \mu q \qquad (3)$$

where w_x represents the pounds of dry solid deposited per square foot in the first x feet, dw_x pounds of dry solid per square foot in distance dx, and dp_x the hydraulic pressure drop. Since $dp_x = -dp_s$, Equation (3) may be rewritten as

$$g_c \frac{dp_s}{dw_x} = \alpha_x \mu q \qquad (4)$$

In general α_x is related experimentally to p_s (1, 2, 4) or may be calculated by empirical or theoretical equations (7).

Equation (4) may be rearranged and integrated to give

$$\int_0^w dw_x = w = \frac{g_c}{\mu q}\int_0^{p-p_1} \frac{dp_s}{\alpha_x} \qquad (5)$$

The pressure p_1 is given by

$$g_c p_1 = \mu R_m q \qquad (6)$$

Equation (5) relates the total cake mass w to the applied pressure p. As a rule it is more convenient to deal with filtrate volume v than it is with dry mass of cake w. If the variables are to be changed, it is necessary to use the material balance

$$w = \frac{s\rho}{1 - ms} v \qquad (7)$$

Since m changes throughout the cake and with time, Equation (7) is generally used only as an approximation (7) with m considered constant. Replacing w in (5) yields

$$v = \frac{g_c(1 - ms)}{\mu s \rho q}\int_0^{p-p_1} \frac{dp_s}{\alpha} \qquad (8)$$

The integration in (8) is carried out at some instant of time when the rate of filtration is q and the applied pressure is p. Although q is a function of p, it is not necessary to have q under the integral sign as a part of the integrand.

In order to introduce time as a variable, it is necessary to use the relations

$$dv = q\,d\theta = \frac{1 - ms}{s\rho} dw \qquad (9)$$

As discussed in a previous paper (7), Equation (9) is valid only if m remains essentially constant and $dm/d\theta$ is virtually zero. This condition is generally fulfilled in filtrations which continue for more than a minute or two.

Equations (8) and (9) are used for determining p vs. θ and v vs. θ relations. In general v would be determined as a function of p in the first step if Equation (8) is utilized. In the second step Equation (9) is employed for finding the v vs. θ relationship based upon a graphical integration of the following equation

$$\theta = \int_0^v \frac{dv}{q} \qquad (10)$$

$$\frac{v_g}{7.48} = \frac{(32.17)(1 - 0.0075)(144)}{(0.001(0.003)(62.4)(0.00226)q_g}\int_0^{\psi-\psi_1} \frac{d\psi_s}{\alpha_x} \qquad (15a)$$

Equations (8) and (10) are the basic formulas necessary for solution of variable-rate–variable-pressure filtrations.

EXAMPLE I.

Talc is to be filtered in a press with a centrifugal pump having the following characteristics:

q_g, gal./(min.)(sq. ft.)	ψ, lb./sq. in.
0	43.5
0.1	42.6
0.2	38.4
0.3	32.3
0.4	24.1
0.5	10.3
0.57	0

Values of specific resistance, α_x, in ft./(lb. mass) are as follows:

α_x, ft./lb. mass	ψ_s, lb./sq. in.
1.057(10[11])	1.10
1.39	2.23
1.97	4.50
2.51	7.9
3.03	11.3
3.67	17.1
4.19	22.7
4.78	28.3
6.46	47.9

Other values necessary to the solution are

μ = 0.001 lb. mass/(ft.)(sec.)
ρ = 62.4 lb. mass/cu. ft.
s = 0.003 fraction solids in slurry
R_m = 2.0(10^{10}) ft.$^{-1}$
m = 2.5 (approximate average value)

The value chosen for the medium resistance is of a reasonable order of magnitude and corresponds to values which might be encountered in practice. It is convenient to change from seconds to minutes, cubic feet to gallons, and pounds force/square foot to pounds force/square inch as follows:

$$\theta = 60\theta_m \qquad (11)$$

$$q = q_g/(60)(7.48) = 0.00226 q_g \qquad (12)$$

$$v = v_g/7.48 \qquad (13)$$

$$p = 144\psi \qquad (14)$$

Substitution in Equation (8) yields

$$v_g = \frac{8.14(10^{10})}{q_g}\int_0^{\psi-\psi_1} \frac{d\psi_s}{\alpha_x} \qquad (15b)$$

The pressure ψ_1 required to overcome the resistance of the medium is obtained from Equation (6); thus

$$\psi_1 = \frac{(0.001)(2)(10^{10})(0.00226)}{(32.17)(144)} q_g \qquad (16a)$$

$$\psi_1 = 9.75 q_g \qquad (16b)$$

Finally Equation (10) becomes

$$\theta_m = \int_0^{v_g} \frac{dv_g}{q_g} \qquad (17)$$

The lowest pressure for which a value of α_x is known is 1.10 lb./sq. in. As it is necessary to determine the area under the $1/\alpha_x$ curve starting at $\psi_s = 0$, an extrapolation as indicated by the dotted lines (Figure 3) is necessary. In Part 2 (7) of this series of papers, the filtration resistance for talc was found to vary as the 0.506 power of ψ_s. Using the values previously determined, one may write α_x as

$$\alpha_x = 8.66(10^{10})\psi_s^{0.506} \qquad (18)$$

This expression is possibly valid down to some low pressure in the neighborhood of 0.1 lb./sq. in. Below this pressure the porosity and specific resistance would approach constant values. For lack of more information and as a reasonable estimate, it will be assumed that α_x approaches its limiting value at 0.1 lb./sq. in. With $\psi_s = 0.1$, α_x in Equation (18) becomes 2.7 (10^{10}) ft./lb. mass, a value that is used in constructing the curves in Figure 3.

The area under the $1/\alpha_x$ curve (Figure 3) between 0 and 1.0 lb./sq. in. is a large fraction of the total area, amounting to about 15% of the area from 0 to 50 lb./sq. in. For materials more compressible than talc, the area between 0 and 1.0 lb./sq. in. could become the major portion of the integral. For filtrations carried out to

TABLE 2

Filtrate volume v_m, gal./sq. ft.	Value of integral θ_m, min. [Equation (17)]	Rate of filtration q_g, gal./(min.)(sq. ft.)	Filtration pressure, lb./sq. in.	Medium pressure, lb./sq. in.
0	10.0	0.540	5.0	5.0
10	19.2	0.500	10.3	4.9
20	41.6	0.420	21.5	4.1
30	69.0	0.322	31.0	3.1
40	103	0.263	35.0	2.6
50	145	0.218	37.5	2.1
60	195	0.186	39.4	1.8
70	253	0.160	40.3	1.6

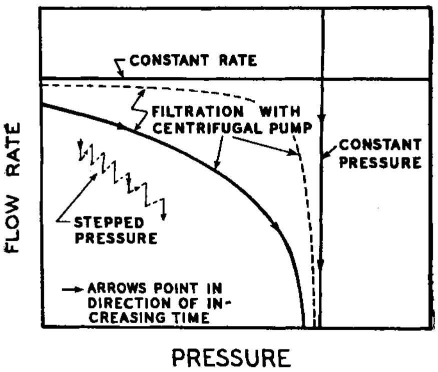

Fig. 1. Flow rate vs. pressure.

pressures of 100 to 150 lb./sq. in., the area under the 0 to 1.0 lb./sq. in. is not so significant for compressibility coefficients, n, less than 0.5 to 0.7. However, for low-pressure filtrations as effected by rotary vacuum operation, the area under the 0 to 1.0 lb./sq. in. portion of the curve might become a significant portion of the total area. While it is difficult to obtain compression-permeability data at very low pressures, it is important to carry experiments to as low a pressure as possible to minimize the need for excessive extrapolation.

In order to obtain v_g in (15*b*) as a function of the pressure ψ, it is necessary to find ψ_1 and q_g for various times. In Figure 4 the pump rate is plotted against both the pressure ψ_1 [Equation (16*b*)] at the entrance to the medium and the filtration pressure ψ in pounds/square inch. At the instant that

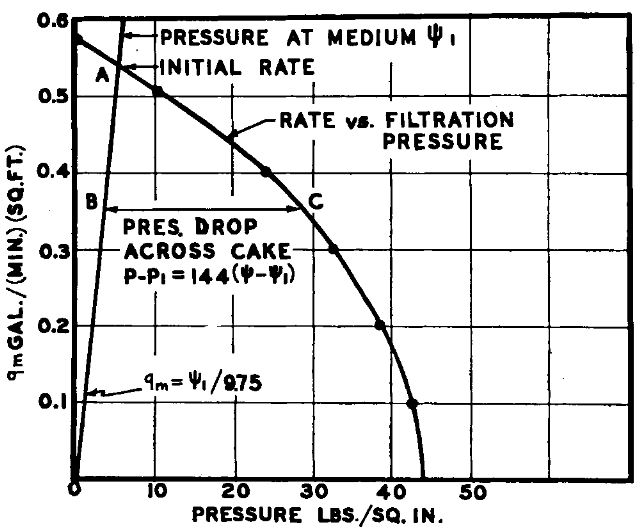

Fig. 4. Pump rate vs. p and p_1.

flow is started, the entire pressure drop will be across the medium, and the initial pressure and rate can be found at *A*, where the two curves intersect. As time progresses, the rate will drop; the pressure ψ at the surface of the cake will increase; and the pressure at the medium ψ_1 will decrease. In general the pressure drop across the cake will be given by lines like *BC*.

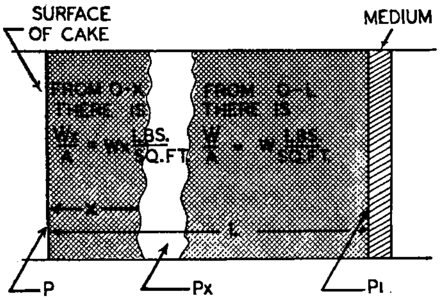

Fig. 2. Schematic diagram of cake.

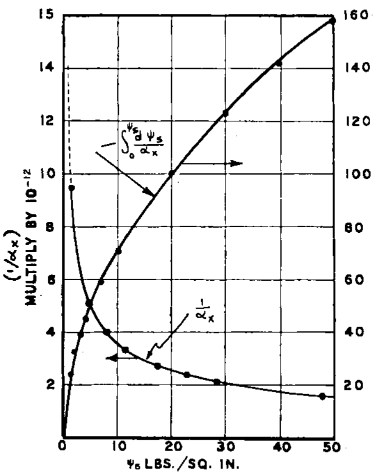

Fig. 3. Graphical integration of Equation (15b).

From Figure 4 q_g, ψ, and ψ_1 can be obtained, and the value of the integral in Equation (15*b*) can be taken from Figure 3. The volume then may be calculated from Equation (15*b*) as illustrated in Table 1.

To complete the problem, the time must be calculated by use of Equation (17) with a graphical integration of a plot of the reciprocal of the rate q_g vs. the volume of filtrate per square foot, v_g. (Figure 5). With values of q_g the pressure can be found from Figure 4 as demonstrated in Table 2.

Plots of the volume per unit area, filtration pressure, and pressure at the medium vs. the time are shown in Figure 6. The medium pressure decreases as the rate drops off. The filtration pressure increases with time and begins to approach a constant pressure toward the end of the curve. The inflection at the first of the curve is caused by the relatively large area under the $1/\alpha_x$ vs. ψ_s curve in the low-pressure region under 1.0 lb./sq. in.

SIMPLIFIED EQUATIONS

A simplified equation can be developed if the filtration resistance is constant and the pump rate is linear with pressure. For practical purposes, average values of resistances can be used over short pressure intervals and, as will be demonstrated, harmonic mean rates may be employed.

If the average resistance is α, integration and rearrangement of Equation (8) leads to

$$\frac{\alpha\mu\rho s}{1 - ms} v = \frac{g_c p - \mu R_m q}{q} = g_c \frac{p}{q} - \mu R_m \qquad (19)$$

if q is linear in p, then the relationship between p and q can be expressed in either of the following ways:

$$q = q_0 - Kp = K(p_m - p) \qquad (20)$$

Equations (19) and (20) combined with Equation (9), $dv = q\, d\theta$, are sufficient for a solution of the p-v-θ relationships. Substitution in Equation (19) for q as presented in Equation (20) will give the v vs. p relation. To obtain a relationship between v and θ, it is convenient first to eliminate p rather than q from Equation (19) as follows:

$$\frac{\alpha\mu\rho s}{1 - ms} v = \frac{g_c\left(\frac{q_0 - q}{K}\right) - \mu R_m q}{q} \qquad (21a)$$

The term $q_0/K = p_m$ represents the maximum pressure (Figure 7) attainable by the pump, if the characteristics are linear, or the intercept of the straight-line approximation on the pressure axis.

$$\frac{\alpha\mu\rho s}{1 - ms} v = g_c\left(\frac{p_m}{q} - \frac{1}{K}\right) - \mu R_m \qquad (21b)$$

Taking the differential yields

$$\frac{\alpha\mu\rho s}{1 - ms} dv = -g_c p_m \frac{dq}{q^2} \qquad (22)$$

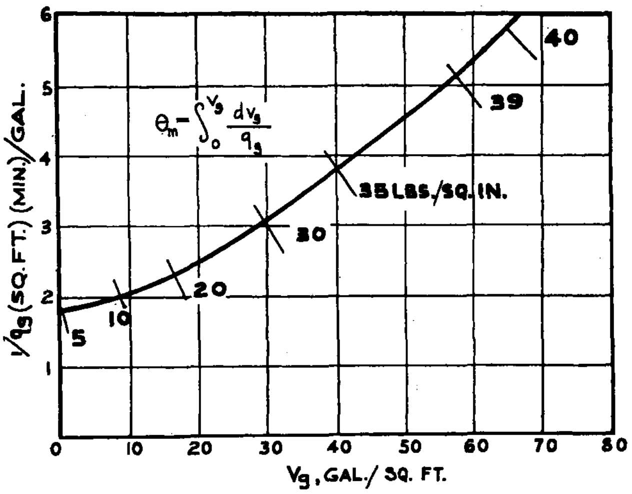

Fig. 5. Graphical integration for determination of time.

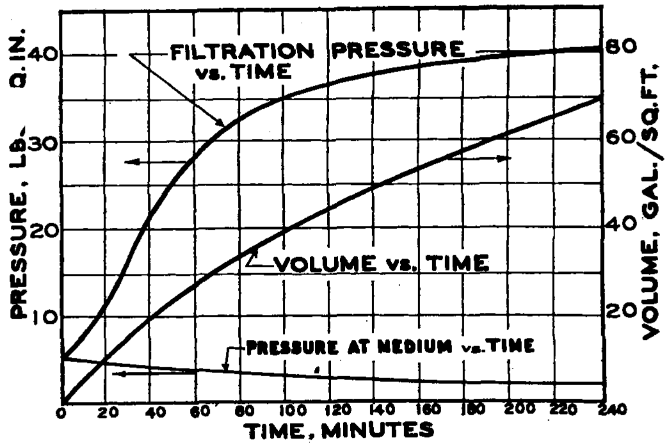

Fig. 6. Pressure and volume vs. time.

Eliminating dv by means of Equation (9), solving for $d\theta$, and placing the proper limits on the integral signs leads to

$$\frac{\alpha\mu\rho s}{1 - ms}\int_{\theta_1}^{\theta} d\theta = -g_c p_m \int_{q_1}^{q} \frac{dq}{q^3} \quad (23)$$

where the rate at the beginning of the period is q_1 as shown in Figure 7. Integration produces

$$\frac{\alpha\mu\rho s}{1 - ms}(\theta - \theta_1) = \frac{g_c p_m}{2}\left(\frac{1}{q^2} - \frac{1}{q_1^2}\right) \quad (24)$$

Substitution for q yields the desired p vs. θ formula; thus

$$\frac{\alpha\mu\rho s}{1 - ms}(\theta - \theta_1) = \frac{g_c p_m}{2}\left[\frac{1}{(q_0 - Kp)^2} - \frac{1}{(q_0 - Kp_{10})^2}\right] \quad (25)$$

when p_{10} is the value of the pressure p_1 at the septum when time is θ_1 and the rate is q_1. From Equation (21*b*) it is possible to obtain $(v - v_1)$ in the following form

$$\frac{\alpha\mu\rho s}{1 - ms}(v - v_1) = g_c p_m\left(\frac{1}{q} - \frac{1}{q_1}\right) \quad (26)$$

The average rate of filtration during the period θ_1 to θ is $\Delta v/\Delta\theta$ or $(v - v_1)/(\theta - \theta_1)$. The latter term can be found by dividing Equation (26) by Equation (24) to produce

$$\frac{\Delta v}{\Delta \theta} = q_{avg} = \frac{2qq_1}{q + q_1} \quad (27)$$

or in reciprocal form

$$\frac{1}{q_{avg}} = \frac{1}{2}\left[\frac{1}{q} + \frac{1}{q_1}\right] \quad (28)$$

Thus q_{avg} is the harmonic mean of q and q_1. Elimination of $1/q$ between Equations (24) and (26) gives

$$\theta - \theta_1 = \frac{\alpha\mu\rho s}{1 - ms}\frac{(v - v_1)^2}{2g_c p_m} + \frac{v - v_1}{q_1} \quad (29)$$

Thus over relatively short pressure ranges where α does not change too much the v vs. θ curves will be parabolic.

Figure 7 presents graphical interpretation of the quantities in Equation (29). The straight line running from q_0 to p_m represents an approximation of the rate vs. pressure curve (dotted). The value q_1 is the rate of filtration at the beginning of the region under consideration, and the pressure p_{10} is the pressure at the medium when the rate is q_1. The values q_1 and p_{10} are related by the equation

$$g_c p_{10} = \mu R_m q_1 \quad (30)$$

If $R_m = 0$, p_{10} is also zero; and q_1 equals q_0, the pump rate at zero pressure. Eliminating q_1 from Equation (24) yields

$$\theta - \theta_1 = \frac{\alpha\mu\rho s}{1 - ms}\frac{(v - v_1)}{2g_c p_m} + \frac{\mu R_m(v - v_1)}{g_c p_{10}} \quad (31)$$

Equation (31) would be identical with the well-known parabolic relation between θ and v in constant pressure filtration [reference *6*, Equation (53)] if p_m and p_{10} were placed equal to the applied filtration pressure. When the medium resistance R_m is zero, Equation (29) reduces to

$$g_c p_m \theta = \left(\frac{\alpha\mu\rho s}{1 - ms}\right)\frac{v^2}{2} + g_c p_m \frac{v}{q_0} \quad (32)$$

when v_1 and θ_1 are placed equal to zero.

ANALYSIS OF EXPERIMENTAL DATA

The experimental determination of filtration resistance involves difficulties whenever constant pressure or constant rate filtration is employed. Although many chemical engineering laboratories are equipped with monte-jus for producing constant-pressure filtration, difficulties are generally encountered in keeping the slurry uniformly suspended in the closed pressure vessel without the characteristics of the precipitate being affected. Constant-rate filtration presents problems with respest to maintaining an absolutely constant rate. The slope of the pressure-time curve in constant-rate filtration varies as the square of the rate, and consequently variations in rate lead to larger errors in calculated values. Although it is possible to utilize either constant-pressure or constant-rate filtration if the proper precautions are observed, variable-pressure-variable-rate operation can be accomplished in a simpler manner with less involved equipment.

Basically all that is needed is a tank with a stirrer connected to a filter press by a pump. While any pumping device can be used, it is essential to avoid changing the nature of the precipitate during the pumping and stirring operation. A high-speed centrifugal pump may develop shearing force on the particles sufficient to cause a breakdown of the particles with resulting increase in filtration resistance. Practically it is difficult to find pumps with sufficiently small capacity for the size of filter convenient to laboratory testing.

If the average specific resistance is defined by

$$\alpha = \frac{p - p_1}{\int_0^{p-p_1} \frac{dp_s}{\alpha_x}} = \frac{p_s}{\int_0^{p_s} \frac{dp_s}{\alpha_x}} \quad (33)$$

Equation (8) becomes

$$v = \frac{(1 - ms)}{\mu s p q} \left[\frac{g_c(p - p_1)}{\alpha} \right] \quad (34)$$

This equation may be placed in any of the following three forms:

$$q = \frac{g_c p}{\mu\left(\frac{s\rho}{1 - ms} v\alpha + R_m\right)} = \frac{g_c p}{\mu(\alpha w + R_m)} \quad (35)$$

$$\frac{g_c}{\mu} \frac{p}{q} = \alpha w + R_m \quad (36)$$

$$g_c p = \alpha \mu q v + p_1 \quad (37)$$

Equation (35) is the familiar form of the filtration equation. Either Equation (36) or (37) may serve as the basis for determining α. Equation (37) is satisfactory if R_m is so small that p_1 can be neglected. In general Equation (36) is the best form to use.

As illustrated in Figure 8, the term $g_c p/\mu q$ can be plotted against the value of w (*5*). In accordance with Equation (36), the average value of the filtration resistance α is given by

$$\alpha = \frac{(g_c p/\mu q) - R_m}{w} \quad (38)$$

which is equivalent to α being the tangent of angle *ABC* in Figure 8. An empirical relationship between the average α and the pressure $p - p_1 = p_s$ can be obtained from an analysis of the graph in accordance with Equation (38). Having the α vs. $p - p_1 = p_s$ data, one can derive the point filtration resistance α_x as a function of p_s rather than having to resort to its experimental determination in a permeability-compression cell. Rearranging Equation (33) gives

$$\int_0^{p_s} \frac{dp_s}{\alpha_x} = \frac{p_s}{\alpha} \quad (39)$$

Differentiating with respect to p_s yields

$$\frac{dp_s}{\alpha_x} = \frac{\alpha\, dp_s - p_s\, d\alpha}{\alpha^2} \quad (40)$$

Solving for α_x results in

$$\alpha_x = \frac{\alpha}{1 - \frac{p_s\, d\alpha}{\alpha\, dp_s}} = \frac{\alpha}{1 - \frac{d \ln \alpha}{d \ln p_s}}$$

If a plot of ln α vs. ln p_s yields a straight line of slope n, Equation (41) reduces to

$$\alpha_x = \frac{\alpha}{1 - n} \quad (42)$$

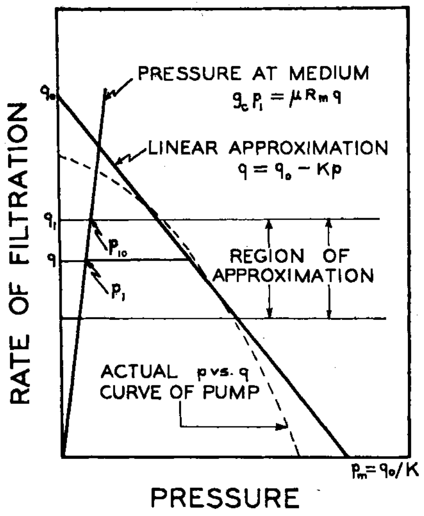

Fig. 7. Linear approximation of pump characteristics.

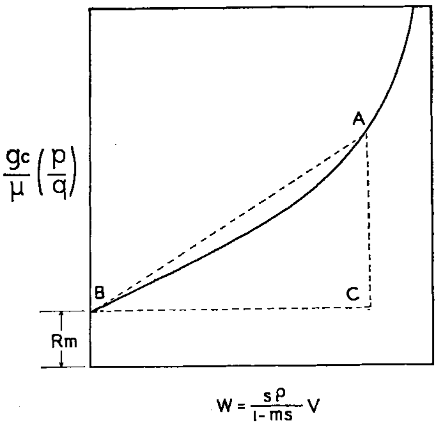

Fig. 8. Graph for determination of average α.

Equation (42) is a satisfactory approximation when n is less than approximately 0.5 to 0.7.

NOTATION

- g_c = conversion factor, poundal/lb. force, (lb. mass) (ft.)/(lb. force) (sec.^{2})
- L = cake thickness, ft.
- K = absolute value of slope of rate vs. pressure curve, Equation (20), cu. ft./(sq. ft.) (sec.) (lb. force)
- m = ratio of mass of wet to mass of dry cake, dimensionless
- n = compressibility factor, slope of ln α vs. ln p_s plot, dimensions meaningless
- p = applied filtration pressure, lb. force/sq. ft. (Previously the author used P for pressure in pounds force/square feet and p for pressure expressed as pounds force/square inches. A change was made in this paper to conform with the A.I.Ch.E. standards for nomenclature presented by Mott Souders in *Chem. Eng. Progr.*, **52**, 255 (1956).)
- p_m = maximum pressure delivered by pump, lb. force/sq. ft.
- p_s = solid compressive pressure at distance x from surface of cake, also total compressive pressure, lb. force/sq. ft.
- p_x = hydraulic pressure at distance x from surface of cake, lb. force/sq. ft.
- p_1 = pressure at interface of medium and cake, lb. force/sq. ft.
- p_{10} = value of p_1 when time equals θ_1, lb. force/sq. ft.
- q = rate of filtration, cu. ft./(sq. ft.) (sec.)
- q_{avg} = harmonic mean rate defined by Equation (28), cu. ft./(sq. ft.) (sec.)
- q_g = rate of filtration, gal./(sq. ft.) (min.)
- q_{10} = rate of filtration when time equals θ_1, cu. ft./(sq. ft.)(sec.)
- q_0 = rate of filtration when pump pressure is zero (Figure 7), cu. ft./(sq. ft.)(sec.)
- R_m = medium resistance, ft.$^{-1}$
- s = fraction solids in slurry, dimensionless
- v = volume of filtrate, cu. ft./sq. ft.
- v_g = volume of filtrate, gal./sq. ft.
- v_1 = volume of filtrate when time equals θ_1, cu. ft./sq. ft.
- x = distance from surface of cake, ft.
- w = total mass of dry solids per unit area, lb. mass/sq. ft.
- w_x = mass of solids per unit area in distance x from surface of cake, lb. mass/sq. ft.
- α = average specific resistance defined by Equation (33), ft./lb. mass
- α_x = value of specific resistance at distance x from cake surface and where solid compressive pressure is p_s, ft./lb. mass
- θ = time, sec.
- θ_m = time, min.
- θ_1 = arbitrary time, sec.
- μ = viscosity, lb. mass/(ft.)(sec.)
- ρ = density, lb. mass/cu. ft.
- ψ = pressure, lb. force/sq. in.
- ψ_s = solid compressive pressure, lb. force/sq. in.
- ψ_1 = pressure at interface between cake and medium, lb. force/sq. in.

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